Lab: AP Review Sheets — AP Physics

Chapter 15: Oscillation — by Hadi Ammar

Background/summary:

This review delves into mechanical oscillation. Whether it's the swinging of a pendulum or the vibrations of a spring, oscillation permeates various natural phenomena. Throughout this review, we'll explore the mathematics, dynamics, and applications of oscillation, empowering you to tackle related problems with confidence on your dreaded AP Physics test.

Oscillation Formulae

$\sum F_x = ma_x$

$F_s = -k\Delta\vec{x}$

$x = A\cos(\omega t + \phi)$

$v_{max} = \omega A$

$a_{max} = \omega^2 A$

$T = 1/\nu$

$\frac{d^2x}{dt^2} + (k)x = 0$, for simple harmonic motion

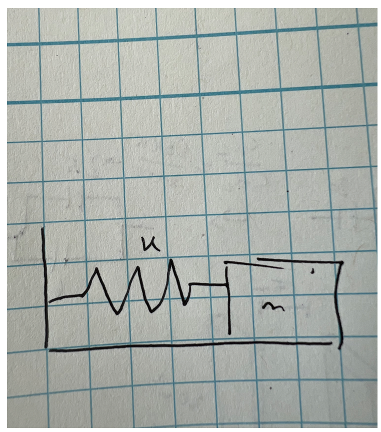
$\omega = (k)^{1/2}$

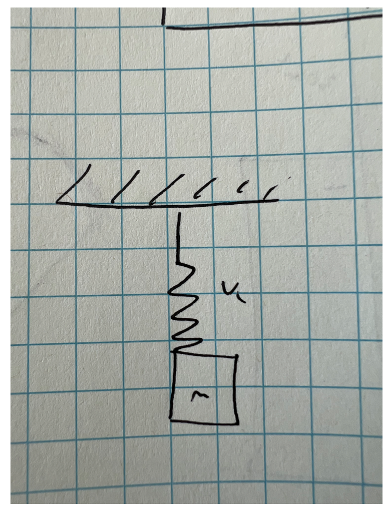
$\omega = 2\pi\nu$

$\omega = \left(\frac{k}{m}\right)^{1/2}$

$E = \frac{1}{2}kA^2$

Oscillation-related Diagrams





What is Oscillation?

Oscillation refers to repetitive back-and-forth motion around an equilibrium point, characterized by periodicity, where motion repeats at regular intervals. Mathematically, oscillation is often described using sinusoidal functions(sine and cosine), which represent the periodic nature of this motion.

Trig again? Are you serious?

Unfortunately, yes. Here's how it works: if we want to be able to characterize oscillatory motion, we must understand what each component o these equations do. Here's the final equation again, and I'm going to explain what each portion does.

$$x = A\left(\cos\left(\omega t + \phi\right)\right)$$

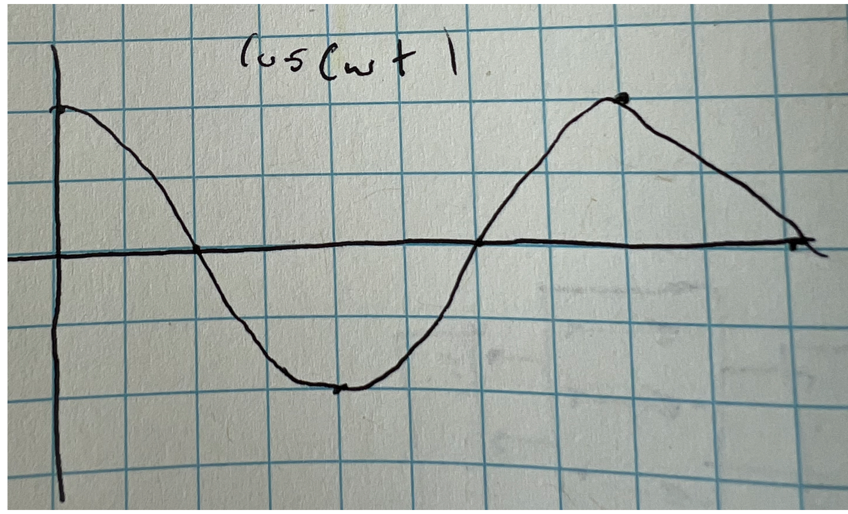
ϕ represents the phase-shift of the motion, this gives us the ability to start the equation at any time throughout the motion.

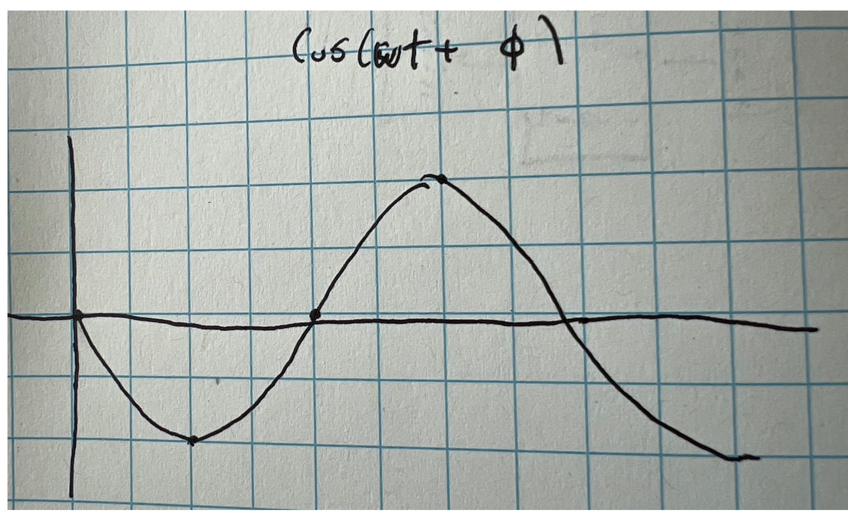
T refers to time obviously

A refers to the maximum amplitude of the motion, this allows us to use this equation for instances where the maximum displacement is no 1.

ω refers to the angular velocity of the objects motion, which can be translated to normal velocity using the $v = r\omega$ equation

Here's how this looks like in a graph:

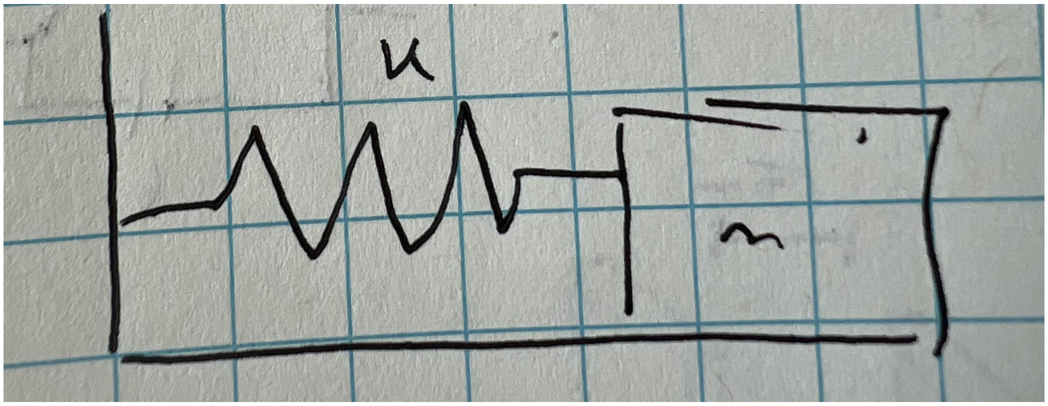




The phase shift here causes the equation to be sifted off it's original starting position

Lets Do Some Problems!

Level 1 (Easy): Derive the energy in a spring system

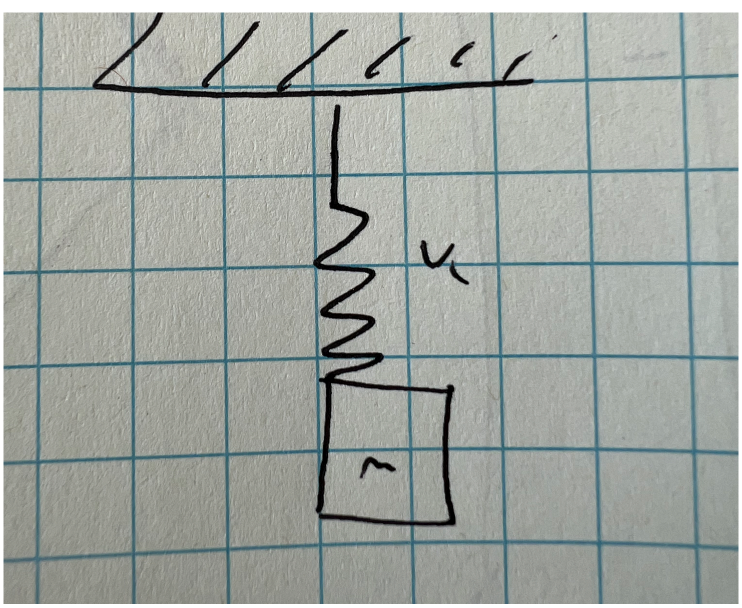


This can be derived most easily when considering the point at which the block of mass M is at A, which means it has 0 velocity but max displacement(its at the extent of its oscillation(the amplitude))

In that case…

$E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}kA^2$ or, after noting that v=0, $E_{total} = \frac{1}{2}kA^2$

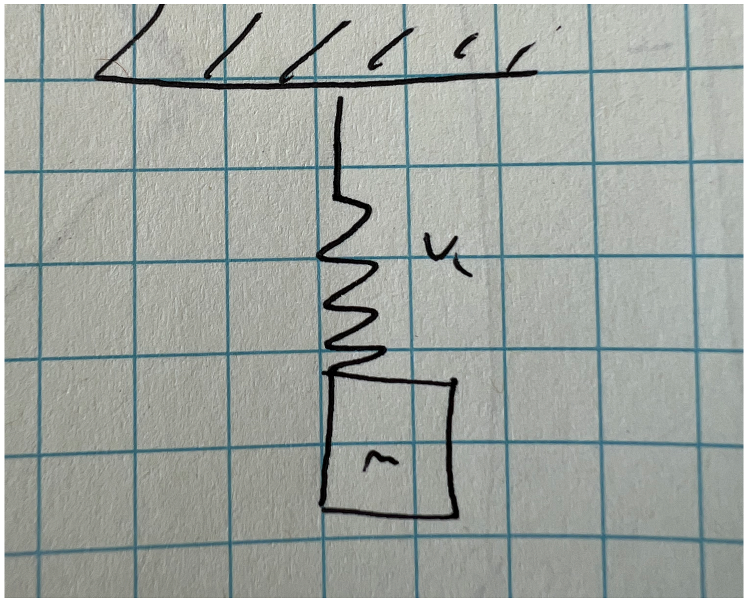
Level 2 (Medium): A spring with spring constant k is hung from the ceiling and with a mass m of .4 kg is attached to it. It is allowed to gently settle until it comes to rest at a point .6 meters below its free-hanging position. The mass is then pulled an additional .2 meters down and released, what is the spring constant?



$$k = \frac{F}{x} = \frac{mg}{d}$$

$$\frac{(.4)(9.8)}{(.6)} = 6.53\ \frac{N}{m}$$

Level 3 (Hard): Consider a spring of spring constant k that has a mass m attached to it. What happens to the spring constant if: A You cut the spring in half? B you double the spring's length?



The best way to approach this problem is by using energy, here is the Potential Energy function for spring stretched from its equilibrium position.

$$U = \frac{1}{2}kd^2$$

Now just assume that there are two half-length strings on top of each other so that the energy wont change

$$U = \frac{1}{2}kd^2 = \frac{1}{2}k_1\left(\frac{d}{2}\right)^2 + \frac{1}{2}k_1\left(\frac{d}{2}\right)^2$$

$$kd^2 = 2k_1\left(\frac{d^2}{4}\right)$$

$$k_1 = 2k$$

If you double the springs length… assuming that there are two full length string on top of each other

$$U = \frac{1}{2}kd^2 = \frac{1}{2}k_1\left(d\right)^2 + \frac{1}{2}k_1\left(d\right)^2$$

$$kd^2 = 2k_1\left(d^2\right)$$

$$k_1 = \frac{k}{2}$$